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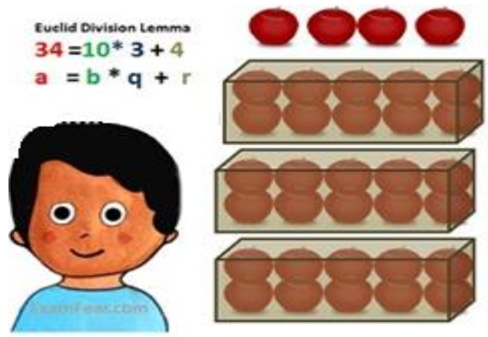
Euclid's Division Lemma:

A **lemma** is a proven statement used for proving another statement.

Theorem 1: "Given positive integers a & b, there exist unique integers q & r satisfying a = b*q + r, $0 \le r < b$".

E.g. let's assume that you have 34 apples & you have box that can accommodate 10 apples, then you have put these Apple in 3 boxes & you will have 4 apples remaining. Also note that the remaining apples are less than box size that is 10.



34 Apples = 10 Apples * 3 Box + 4 Apples

If you compare this with Euclid's Division Lemma

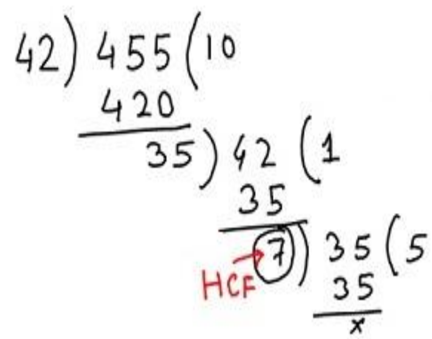
a = b*q + r, $0 \le r < b$,

Then a = 34, b=10, q=3 & r =4, $0 \le 4 < 10$

Euclid's division algorithm is a technique to compute the Highest Common Factor (HCF) of two given positive integers.

HCF of two positive integers a and b is the largest positive integer d that divides both a and b

Let's find HCF of the integers 455 and 42.



We start with the larger integer, that is, 455. Then we use Euclid's lemma to get

- $455 = 42 \times 10 + 35$

Now consider the divisor 42 and the remainder 35, and apply the division lemma to get

- $42 = 35 \times 1 + 7$

Now consider the divisor 35 and the remainder 7, and apply the division lemma to get

- $35 = 7 \times 5 + 0$

Notice that the remainder has become zero, and we cannot proceed any further. We claim that the HCF of 455 and 42 is 7.

Numerical: Show that every positive even integer is of the form 2q, and that every positive odd integer is of the form 2q + 1, where q is some integer.

Solution: Let a be any positive integer and b = 2. Then, by Euclid's algorithm, a = 2q + r, for some

integer $q \ge 0$, and r = 0 or r = 1, because $0 \le r < 2$. So, a = 2q or 2q + 1.

If a is of the form 2q, then a is an even integer. Also, a positive integer can be either even or odd.

Therefore, any positive odd integer is of the form 2q + 1.

Numerical: A sweet seller has 420 kaju barfis and 130 badam barfis. She wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the number of that can be placed in each stack for this purpose?

Solution: This can be done by trial and error. But to do it systematically, we find HCF (420, 130). Then this number will give the maximum number of barfis in each stack and the number of stacks will then be the least. The area of the tray that is used up will be the least.
Now, let us use Euclid's algorithm to find their HCF. We have :

$420 = 130 \times 3 + 30$

$130 = 30 \times 4 + 10$

$30 = 10 \times 3 + 0$

So, the HCF of 420 and 130 is 10.

Therefore, the sweet seller can make stacks of 10 for both kinds of barfi.

Fundamental Theorem of Arithmetic:

Given by given by Carl Friedrich Gauss, it states that every composite number can be written as the product of powers of primes E.g.: 30 = 2* 3* 5

Theorem 2 : Every composite number can be expressed as a product of primes, and this factorization is **unique**, apart from the order in which the prime factors occur.

The prime factorization of a natural number is unique, except for the order of its factors.

E.g. 30 = 2* 3* 5 = 2* 5 * 3 = 3 * 2 * 5 = 3 * 5 * 2 = 5 * 3 * 2 = 5 * 2 * 3

Prime factorization of 30 is unique; it has number 2, 3 & 5 ignoring the order.

Numerical: Consider the numbers 4^n, where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.

Solution: If the number 4^n, for any n, were to end with the digit zero, then it would be divisible by 5. But as per factorization theorem, 4^n has the only prime factor 2 , so it is not divisible by 10.

Numerical: Find the LCM and HCF of 6 and 20 by the prime factorisation method.

Solution: We have : $6 = 2^1 \times 3^1$ and $20 = 2 \times 2 \times 5 = 2^2 \times 5^1$.
Note that HCF $(6, 20) = 2^1 = 2$
HCS is Product of the smallest power of each common prime factor in the numbers.

LCM $(6, 20) = 2^2 \times 3^1 \times 5^1 = 60$
LCM is Product of the greatest power of each prime factor, involved in the numbers.